# Math 43 Midterm 1 Review

## CONICS

[1] Find the equations of the following conics.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its focus, vertex, directrix & axis of symmetry.
If the equation corresponds to an ellipse, find its center, foci & the endpoints of the major and minor axes.
If the equation corresponds to a hyperbola, find its center, foci, vertices & asymptotes.

| | | |
|---|---|---|
| [a] | Ellipse: | foci $(-5, 2)$ and $(3, 2)$, minor axis of length $12$ |
| [b] | Parabola: | vertex $(-4, 6)$, directrix $x = 16$ |
| [c] | Hyperbola: | one focus $(2, 1)$, asymptotes $y = 3x - 7$ and $y = -3x + 5$ |
| [d] | Ellipse: | endpoints of major axis $(-7, -3)$ and $(5, -3)$, endpoints of minor axis $(-1, -7)$ and $(-1, 1)$ |
| [e] | Hyperbola: | vertices $(4, -1)$ and $(-2, -1)$, passing through $(5, 6)$ |
| [f] | Parabola: | vertex $(-4, -6)$, focus $(-4, -11)$ |
| [g] | Ellipse: | vertices $(-3, -7)$ and $(-3, 5)$, foci $(-3, -6)$ and $(-3, 4)$ |
| [h] | Circle: | endpoints of diameter $(-8, 7)$ and $(-2, -3)$ |
| [i] | Hyperbola: | vertices $(-3, -7)$ and $(-3, 5)$, foci $(-3, -9)$ and $(-3, 7)$ |
| [j] | Ellipse: | foci $(5, -2)$ and $(5, 4)$, major axis of length $12$ |
| [k] | Parabola: | focus $(4, -6)$, directrix $y = 16$ |
| [l] | Ellipse: | vertices $(12, 1)$ and $(2, 1)$, minor axis of length $4$ |
| [m] | Hyperbola: | one vertex $(6, 2)$, asymptotes $y = 2x - 4$ and $y = -2x + 8$ |
| [n] | Parabola: | vertex at origin, horizontal axis of symmetry, passing through $(-8, -7)$ |
| [o] | Ellipse: | endpoints of minor axis $(3, 11)$ and $(3, 7)$, major axis of length $10$ |

[2] Determine if each equation corresponds to a circle, a parabola, an ellipse or a hyperbola.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its focus, vertex, directrix & axis of symmetry.
If the equation corresponds to an ellipse, find its center, foci & the endpoints of the major and minor axes.
If the equation corresponds to a hyperbola, find its center, foci, vertices & asymptotes.

[a] $9x^2 - 16y^2 - 36x + 32y + 164 = 0$

[b] $y^2 + 8x + 2y - 39 = 0$

[c] $x^2 + y^2 + 6x - 8y + 21 = 0$

[d] $9x^2 + 4y^2 + 72x + 40y + 208 = 0$

[3] Draw diagrams and write algebraic equations involving distances to answer the following questions.

[a] A car is traveling along a route so that it is always $40$ miles closer to a certain cell tower than it is to a certain monument. What is the shape of the route ?

[b] A ship is traveling along a route so that any signal originating at Old Port, headed to the ship, and then immediately resent to New Port, always takes $8$ seconds to complete the transmission. What is the shape of the route ?

[c] A satellite is traveling in an orbit so that it is always $5{,}000$ miles from the center of the earth. What is the shape of the orbit ?

## POLAR

[1] Remember that a single point in the plane has infinitely many polar co-ordinates.
Consider the point with polar co-ordinates $(7, \frac{2\pi}{3})$.

[a] Find another pair of polar co-ordinates for this point, using a positive $r-$value, and a positive $\theta-$value.

[b] Find another pair of polar co-ordinates for this point, using a positive $r-$value, and a negative $\theta-$value.

[c] Find another pair of polar co-ordinates for this point, using a negative $r-$value, and a positive $\theta-$value.

[d] Find another pair of polar co-ordinates for this point, using a negative $r-$value, and a negative $\theta-$value.

[2] Convert the following points or equations.

[a] the point with polar co-ordinates $(8, \frac{5\pi}{6})$ to rectangular co-ordinates

[b] the point with rectangular co-ordinates $(-6, -2)$ to polar co-ordinates

[c] the rectangular equation $x^2 - y^2 - 2x = 0$ to polar

[d] the polar equation $r = \dfrac{7}{4 - 2\cos\theta}$ to rectangular

[e] the rectangular equation $3x - 2y + 6 = 0$ to polar

[f] the polar equation $r = \cos 2\theta$ to rectangular

[g] the rectangular equation $x^2 + 6y - 9 = 0$ to polar

[h] the polar equation $\theta = \dfrac{5\pi}{6}$ to rectangular

[3] Run the standard tests for symmetry for the polar equation $r^3 = 1 - \sin 2\theta$, and state the conclusions.
What is the minimum interval of $\theta -$ values that must be plotted before using symmetry to complete the graph ?

[4] Find the values of $\theta \in [0, 2\pi)$ at which the graph of the polar equation $r = 2\cos 2\theta + 1$ passes through the pole.

[5] Name the shape of the graphs of the following polar equations.
If the graph is a rose curve, state the number of petals.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| [a] | $r = 5 - 5\sin\theta$ | [b] | $r = 7\sin 6\theta$ | [c] | $r = 5$ | [d] | $\theta = 7$ |
| [e] | $r = 4\sin 9\theta$ | [f] | $r = 2 + 3\cos\theta$ | [g] | $r = 6 - 4\cos\theta$ | [h] | $r = 3\sin\theta$ |
| [i] | $r = 6 + 2\sin\theta$ | | | | | | |

[6] Determine if each polar equation corresponds to a circle, a parabola, an ellipse or a hyperbola.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its eccentricity, focus, directrix & vertex.
If the equation corresponds to an ellipse, find its eccentricity, foci, directrix, center & the endpoints of the major axes and latera recta.
If the equation corresponds to a hyperbola, find its eccentricity, foci, directrix, center, vertices & the endpoints of the latera recta.
**Do not convert the equations to rectangular co-ordinates.**
**Final answers must be in rectangular co-ordinates.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| [a] | $r = \dfrac{10}{3 - 3\sin\theta}$ | [b] | $r = \dfrac{10}{3 - 2\cos\theta}$ | [c] | $r = \dfrac{10}{2 + 3\sin\theta}$ | [d] | $r = 10$ |

[7] Find the polar equations of the following conics with their focus at the pole.

[a] Parabola: directrix $x = 7$

[b] Parabola: vertex $(7, \frac{3\pi}{2})$

[c] Ellipse: eccentricity $\frac{3}{4}$, directrix $y = 5$

[d] Ellipse: vertices $(4, 0)$ and $(2, \pi)$

[e] Hyperbola: eccentricity $\frac{5}{2}$, directrix $x = -3$

[f] Hyperbola: vertices $(3, \frac{3\pi}{2})$ and $(15, \frac{3\pi}{2})$

[8] Draw diagrams and write algebraic equations involving distances to answer the following questions.

A drinking fountain is $15$ feet from the wall of a school building.

[a] A cat is running on the school grounds, so that it is always three times as far from the wall as it is from the fountain. What is the shape of the cat's path ?

[b] A dog is running on the school grounds, so that it is always three times as far from the fountain as it is from the wall. What is the shape of the dog's path ?

[c] A chicken is running on the school grounds, so that it is always as far from the wall as it is from the fountain. What is the shape of the chicken's path ?

[9] Sketch the graphs of the polar equations in [5][a], [f], [g] and [i] using the shortcut process shown in lecture.
Find all $x -$ and $y -$ intercepts.

# ANSWERS

## CONICS

[1] [a]

| | |
|---|---|
| Center: | $(-1, 2)$ |
| Foci: | GIVEN |
| Endpoints of major axis: | $(-1 \pm 2\sqrt{13}, 2)$ |
| Endpoints of minor axis: | $(-1, -4)$ and $(-1, 8)$ |
| Equation: | $\dfrac{(x+1)^2}{52} + \dfrac{(y-2)^2}{36} = 1$ |

[b]

| | |
|---|---|
| Focus: | $(-24, 6)$ |
| Vertex: | GIVEN |
| Directrix: | GIVEN |
| Axis of Symmetry: | $y = 6$ |
| Equation: | $(y-6)^2 = -80(x+4)$ |

[c]

| | |
|---|---|
| Center: | $(2, -1)$ |
| Foci: | $(2, 1)$ and $(2, -3)$ |
| Vertices: | $(2, -1 \pm \frac{3\sqrt{10}}{5})$ |
| Asymptotes: | GIVEN |
| Equation: | $\dfrac{(y+1)^2}{\frac{18}{5}} - \dfrac{(x-2)^2}{\frac{2}{5}} = 1$ |

[d]

| | |
|---|---|
| Center: | $(-1, -3)$ |
| Foci: | $(-1 \pm 2\sqrt{5}, -3)$ |
| Endpoints of major axis: | GIVEN |
| Endpoints of minor axis: | GIVEN |
| Equation: | $\dfrac{(x+1)^2}{36} + \dfrac{(y+3)^2}{16} = 1$ |

[e]

| | |
|---|---|
| Center: | $(1, -1)$ |
| Foci: | $(1 \pm 6\sqrt{2}, -1)$ |
| Vertices: | GIVEN |
| Asymptotes: | $y + 1 = \pm\sqrt{7}(x-1)$ |
| Equation: | $\dfrac{(x-1)^2}{9} - \dfrac{(y+1)^2}{63} = 1$ |

[f]

| | |
|---|---|
| Focus: | GIVEN |
| Vertex: | GIVEN |
| Directrix: | $y = -1$ |
| Axis of Symmetry: | $x = -4$ |
| Equation: | $(x+4)^2 = -20(y+6)$ |

[g]

| | |
|---|---|
| Center: | $(-3, -1)$ |
| Foci: | GIVEN |
| Endpoints of major axis: | GIVEN |
| Endpoints of minor axis: | $(-3 \pm \sqrt{11}, -1)$ |
| Equation: | $\dfrac{(x+3)^2}{11} + \dfrac{(y+1)^2}{36} = 1$ |

[h] Center: $(-5, 2)$

Radius: $\sqrt{34}$

Equation: $(x+5)^2 + (y-2)^2 = 34$

[i] Center: $(-3, -1)$

Foci: GIVEN

Vertices: GIVEN

Asymptotes: $y+1 = \pm\frac{3\sqrt{7}}{7}(x+3)$

Equation: $\dfrac{(y+1)^2}{36} - \dfrac{(x+3)^2}{28} = 1$

[j] Center: $(5, 1)$

Foci: GIVEN

Endpoints of major axis: $(5, -5)$ and $(5, 7)$

Endpoints of minor axis: $(5 \pm 3\sqrt{3}, 1)$

Equation: $\dfrac{(x-5)^2}{27} + \dfrac{(y-1)^2}{36} = 1$

[k] Focus: GIVEN

Vertex: $(4, 5)$

Directrix: GIVEN

Axis of Symmetry: $x = 4$

Equation: $(x-4)^2 = -44(y-5)$

[l] Center: $(7, 1)$

Foci: $(7 \pm \sqrt{21}, 1)$

Endpoints of major axis: GIVEN

Endpoints of minor axis: $(7, -1)$ and $(7, 3)$

Equation: $\dfrac{(x-7)^2}{25} + \dfrac{(y-1)^2}{4} = 1$

[m] Center: $(3, 2)$

Foci: $(3 \pm 3\sqrt{5}, 2)$

Vertices: $(0, 2)$ and $(6, 2)$

Asymptotes: GIVEN

Equation: $\dfrac{(x-3)^2}{9} - \dfrac{(y-2)^2}{36} = 1$

[n] Focus: $\left(-\dfrac{49}{32}, 0\right)$

Vertex: GIVEN

Directrix: $x = \dfrac{49}{32}$

Axis of Symmetry: $y = 0$

Equation: $y^2 = -\dfrac{49}{8}x$

[o] Center: $(3, 9)$

Foci: $(3 \pm \sqrt{21}, 9)$

Endpoints of major axis: $(8, 9)$ and $(-2, 9)$

Endpoints of minor axis: GIVEN

Equation: $\dfrac{(x-3)^2}{25} + \dfrac{(y-9)^2}{4} = 1$

[2] [a] HYPERBOLA

Center: $(2, 1)$

Foci: $(2, -4)$ and $(2, 6)$

Vertices: $(2, -2)$ and $(2, 4)$

Asymptotes: $y - 1 = \pm \frac{3}{4}(x-2)$

[b] PARABOLA

Focus: $(3, -1)$

Vertex: $(5, -1)$

Directrix: $x = 7$

Axis of Symmetry: $y = -1$

[c] CIRCLE

Center: $(-3, 4)$

Radius: 2

[d] ELLIPSE

Center: $(-4, -5)$

Foci: $(-4, -5 \pm \sqrt{5})$

Endpoints of major axis: $(-4, -8)$ and $(-4, -2)$

Endpoints of minor axis: $(-6, -5)$ and $(-2, -5)$

[3] [a] (part of) hyperbola [b] ellipse [c] circle

## POLAR

[1] [a] $(7, \frac{8\pi}{3})$ [b] $(7, -\frac{4\pi}{3})$ [c] $(-7, \frac{5\pi}{3})$ [d] $(-7, -\frac{\pi}{3})$

[2] [a] $(-4\sqrt{3}, 4)$ [b] $(2\sqrt{10}, 3.46)$

[c] $r = \dfrac{2\cos\theta}{\cos 2\theta} = 2\cos\theta \sec 2\theta$ [d] $12x^2 + 16y^2 - 28x - 49 = 0$

[e] $r = \dfrac{6}{2\sin\theta - 3\cos\theta}$ [f] $(x^2 + y^2)^3 = (x^2 - y^2)^2$

[g] $r = \dfrac{3}{1+\sin\theta}\, or \dfrac{-3}{1-\sin\theta}$ [h] $y = -\frac{\sqrt{3}}{3}x$

[3] Symmetry over polar axis: substituting $(r, -\theta)$ gives $r^3 = 1 + \sin 2\theta$ no conclusion

substituting $(-r, \pi - \theta)$ gives $r^3 = -1 - \sin 2\theta$ no conclusion

Symmetry over pole: substituting $(-r, \theta)$ gives $r^3 = -1 + \sin 2\theta$ no conclusion

substituting $(r, \pi + \theta)$ gives $r^3 = 1 - \sin 2\theta$ symmetric over pole

Symmetry over $\theta = \frac{\pi}{2}$: substituting $(-r, -\theta)$ gives $r^3 = -1 - \sin 2\theta$ no conclusion

substituting $(r, \pi - \theta)$ gives $r^3 = 1 + \sin 2\theta$ no conclusion

Minimum interval $\theta \in [0, \pi]$ or $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

[4] $0 = 2\cos 2\theta + 1$ for $0 \le \theta < 2\pi$

$\cos 2\theta = -\frac{1}{2}$

$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ since $0 \le 2\theta < 4\pi$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

[5] [a] cardioid [b] rose curve with 12 petals [c] circle [d] line
[e] rose curve with 9 petals [f] limacon with inner loop [g] limacon with dimple [h] circle
[i] convex limacon

[6] [a] PARABOLA

Eccentricity: 1
Focus: $(0, 0)$
Directrix: $y = -\frac{10}{3}$
Vertex: $(0, -\frac{5}{3})$

[b] ELLIPSE

Eccentricity: $\frac{2}{3}$
Foci: $(0, 0)$ and $(8, 0)$
Directrix: $x = -5$
Center: $(4, 0)$
Endpoints of major axis: $(-2, 0)$ and $(10, 0)$
Endpoints of latera recta: $(0, \pm\frac{10}{3})$ and $(8, \pm\frac{10}{3})$

[c] HYPERBOLA

Eccentricity: $\frac{3}{2}$
Foci: $(0, 0)$ and $(0, 12)$
Directrix: $y = \frac{10}{3}$
Center: $(0, 6)$
Vertices: $(0, 2)$ and $(0, 10)$
Endpoints of latera recta: $(\pm 5, 0)$ and $(\pm 5, 12)$

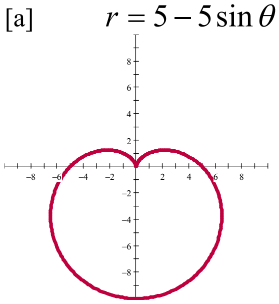
[d] Center: $(0, 0)$
Radius: 10

[7] [a] $r = \dfrac{7}{1+\cos\theta}$ [b] $r = \dfrac{14}{1-\sin\theta}$ [c] $r = \dfrac{15}{4+3\sin\theta}$

[d] $r = \dfrac{8}{3-\cos\theta}$ [e] $r = \dfrac{15}{2-5\cos\theta}$ [f] $r = \dfrac{15}{2-3\sin\theta}$

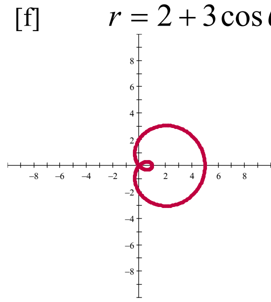
[8] [a] ellipse [b] (part of) hyperbola [c] parabola

[9] [a] $r = 5 - 5\sin\theta$



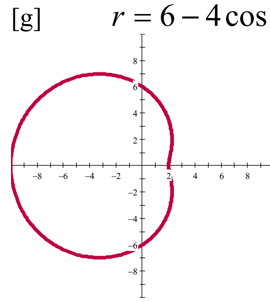
$(\pm 5, 0)$, $(0, 0)$, $(0, -10)$

[f] $r = 2 + 3\cos\theta$



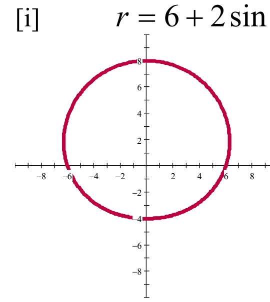
$(5, 0)$, $(1, 0)$, $(0, 0)$, $(0, \pm 2)$

[g] $r = 6 - 4\cos\theta$



$(2, 0)$, $(-10, 0)$, $(0, \pm 6)$

[i] $r = 6 + 2\sin\theta$



$(\pm 6, 0)$, $(0, 8)$, $(0, -4)$